

# Feasibility of extracting a $\Sigma^-$ admixture probability in the neutron-rich $^{10}_{\Lambda}\text{Li}$ hypernucleus

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## Abstract

We theoretically examine production of the neutron-rich  $^{10}_{\Lambda}\text{Li}$  hypernucleus by a double-charge exchange ( $\pi^-$ ,  $K^+$ ) reaction on a  $^{10}\text{B}$  target with distorted-wave impulse approximation calculations. We calculate the inclusive spectrum at the incident momentum 1.20 GeV/c by a one-step mechanism  $\pi^- p \rightarrow K^+ \Sigma^-$  via  $\Sigma^-$  doorways caused by a  $\Sigma^- p \leftrightarrow \Lambda n$  coupling. The resultant spectrum can explain the magnitude of the recent experimental data, so that the  $\Sigma^-$  admixture probability in  $^{10}_{\Lambda}\text{Li}$  is found to be the order of  $10^{-1}$  %. The ( $\pi^-$ ,  $K^+$ ) reaction provides a capability of extracting properties of wave functions with  $\Lambda$ - $\Sigma$  coupling effects in neutron-rich nuclei, as well as the reaction mechanism.

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It has been discussed that a study of strangeness in nuclei would provide new information on nuclear physics and astrophysics [1]. The presence of hyperons in high-density nuclear medium significantly affects the maximal mass of neutron stars and compact stars, because it makes the Equation of State (EoS) soften [2]. The negatively charged  $\Sigma^-$  hyperon was expected to play an essential role in description of neutron stars, whereas the baryon fraction is found to depend on properties of hypernuclear potentials in neutron stars. On the other hand, several theoretical studies [3] suggested that a repulsive component in  $\Sigma^-$ -nucleus potentials is needed to reproduce the observed spectra of  $(\pi^-, K^+)$  reactions on nuclear targets [4] and also the strong level-shifts and widths of the  $\Sigma^-$  atomic  $X$ -ray data. This repulsion originates from the  $\Sigma N$   $T = 3/2$ ,  $^3S_1$  channel which corresponds to a quark Pauli-forbidden state in the baryon-baryon system [5]. However, since a strong  $\Sigma^- p \rightarrow \Lambda n$  conversion occurs at a nuclear surface, it is difficult to extract the nature of the  $\Sigma^-$  hyperon in nuclear medium from such experimental data on nuclear targets.

One of the most promising subjects to examine the hypernuclear potential in a neutron-excess environment is a study of neutron-rich  $\Lambda$  hypernuclei [6]. The  $\Lambda$  hyperon in nuclei is known to act as a nuclear “glue”, and can often make the system bound even if a core-nucleus is unbound, e.g.,  $^6_\Lambda\text{He}$ . In addition, it is suggested that in  $s$ -shell  $\Lambda$  hypernuclei an attractive mechanism appears due to the  $\Lambda$ - $\Sigma$  coupling which is related to a three-body  $\Lambda NN$  force [7], and their  $\Sigma$ -mixing probabilities are 1-2 %, as discussed in few-body calculations [8]. This situation is found to be more coherently enhanced in the neutron-excess environment [9]. Therefore, we believe that there are a lot of exotic neutron-rich  $\Lambda$  hypernuclei beyond the neutron-drip line.

# ——— **FIG. 1** ———

The experimental attempts to produce neutron-rich  $\Lambda$  hypernuclei were carried out by reactions based on a double-charge exchange (DCX) mechanism, as  $(K_{\text{Stopped}}^-, \pi^+)$  [10, 11] and  $(\pi^-, K^+)$  [12]. Further experiments in the nuclear  $(\pi^-, K^+)$  reactions are also planned at J-PARC [13]. The production of the neutron-rich  $\Lambda$  hypernuclei by the DCX reaction  $(\pi^-, K^+)$  would conventionally proceed by a two-step mechanism of the meson charge-exchange,  $\pi^- p \rightarrow \pi^0 n$  followed by  $\pi^0 p \rightarrow K^+ \Lambda$ , as shown in Fig. 1(a), or  $\pi^- p \rightarrow K^0 \Lambda$  followed by  $K^0 p \rightarrow K^+ n$ . Another exotic mechanism is a one-step process,  $\pi^- p \rightarrow K^+ \Sigma^-$  via  $\Sigma^-$  doorways caused by the  $\Sigma^- p \leftrightarrow \Lambda n$  coupling in  $\Lambda$  hypernuclei, as shown in Fig. 1(b). Tretyakova and

Lanskoy [14] theoretically found that the two-step mechanism in the  $^{10}\text{B}(\pi^-, K^+)$  reaction is more dominant compared to the one-step one, where the  $\Sigma^-$  admixture probability is as small as the order of  $10^{-2}$  %. Thus they claimed that the magnitude of the cross section of the  $^{10}_{\Lambda}\text{Li}$  bound state in the  $(\pi^-, K^+)$  reaction is as large as 38-67 nb/sr at the incident momentum  $p_\pi = 1.05$  GeV/c ( $0^\circ$ ), where the cross section of the conventional  $(\pi^+, K^+)$  reaction on nuclear targets is at its maximum [15].

Recently, Saha et al. [12] have performed the first measurement of a significant yield for the  $^{10}_{\Lambda}\text{Li}$  hypernucleus in  $(\pi^-, K^+)$  reactions on a  $^{10}\text{B}$  target, whereas no clear peak has been observed with the lack of the experimental statistics. The data show that the absolute cross section for  $^{10}_{\Lambda}\text{Li}$  at 1.20 GeV/c ( $d\sigma/d\Omega \sim 11$  nb/sr) is twice larger than that at 1.05 GeV/c ( $d\sigma/d\Omega \sim 6$  nb/sr). This incident-momentum dependence of  $d\sigma/d\Omega$  exhibits a trend in the opposite direction for the theoretical prediction of Ref. [14]. This might mean that the one-step mechanism is favored rather than the two-step mechanism, as pointed out in Ref. [12].

In this paper, we theoretically investigate production of the neutron-rich  $^{10}_{\Lambda}\text{Li}$  hypernucleus by the DCX  $(\pi^-, K^+)$  reaction on a  $^{10}\text{B}$  target at 1.20 GeV/c, within a distorted-wave impulse approximation (DWIA). In order to understand the mechanism of this reaction, we focus on the  $\Lambda$  spectrum populated by the one-step mechanism,  $\pi^- p \rightarrow K^+ \Sigma^-$  via  $\Sigma^-$  doorways due to the  $\Sigma^- p \leftrightarrow \Lambda n$  coupling. This is the first attempt to extract the probability of the  $\Sigma^-$  admixture in a neutron-rich  $\Lambda$  hypernucleus from available experimental data phenomenologically. We also discuss a small contribution of the two-step processes in the  $(\pi^-, K^+)$  reactions within the eikonal approximation.

Now let us consider the DCX  $(\pi^-, K^+)$  reaction on the  $^{10}\text{B}$  target within the DWIA. In order to fully describe the one-step process via  $\Sigma^-$  doorways, as shown in Fig. 1(b), we perform a  $\Lambda$ - $\Sigma$  coupled-channel calculation [16], evaluating the production cross section of  $\Lambda$  hypernuclear states in  $^{10}_{\Lambda}\text{Li}$ . We assume a two-channel coupled wave function for simplicity, which is given by

$$|^{10}_{\Lambda}\text{Li}\rangle = \varphi_{\Lambda}(\mathbf{r}) |^9\text{Li} \otimes \Lambda\rangle + \varphi_{\Sigma}(\mathbf{r}) |^9\text{Be}^* \otimes \Sigma^-\rangle, \quad (1)$$

where  $\langle\varphi_{\Lambda}|\varphi_{\Lambda}\rangle + \langle\varphi_{\Sigma}|\varphi_{\Sigma}\rangle = 1$ , and  $\mathbf{r}$  denotes a relative coordinate between the core-nucleus and the hyperon. The probability of the  $\Sigma^-$  admixture in the  $\Lambda$  hypernucleus can be obtained by  $P_{\Sigma^-} = \langle\varphi_{\Sigma}|\varphi_{\Sigma}\rangle$ . It should be noticed that the core-excited state ( $^9\text{Be}^*$ ) in the  $\Sigma^-$  channel is assumed to be a one effective state which can be fully coupled with the  $^9\text{Li}$  core state

via the  $\Lambda$ - $\Sigma$  coupling, rather than the  ${}^9\text{Be}(\frac{3}{2}^-; \frac{1}{2})$  ground state. Thus, we assume  $\Delta M = 80$  MeV effectively for the threshold-energy difference between  ${}^9\text{Li} + \Lambda$  and  ${}^9\text{Be}^* + \Sigma^-$  channels. The mixed  $\Sigma^-$ -hyperon in  ${}^{10}_\Lambda\text{Li}$  is regarded as a deeply bound particle in the nucleus, where the  $\pi^- p \rightarrow K^+ \Sigma^-$  transition takes place under an energy-off-shell condition.

In order to calculate the nuclear  $(\pi^-, K^+)$  spectrum, we employ the Green's function method [17], which is one of the most powerful treatments in a calculation of the spectrum which includes not only bound states but also continuum states with an absorptive potential for spreading components. The complete Green's function  $\mathbf{G}$  describes all information concerning  $({}^9\text{Li} \otimes \Lambda) + ({}^9\text{Be}^* \otimes \Sigma^-)$  coupled-channel dynamics. We obtain it by solving the following equation with the hyperon-nucleus potential  $\mathbf{U}$  numerically:

$$\mathbf{G} = \mathbf{G}^{(0)} + \mathbf{G}^{(0)} \mathbf{U} \mathbf{G}, \quad (2)$$

where

$$\mathbf{G} = \begin{pmatrix} G_{\Lambda\Lambda} & G_{\Lambda\Sigma} \\ G_{\Sigma\Lambda} & G_{\Sigma\Sigma} \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} U_{\Lambda\Lambda} & U_{\Lambda\Sigma} \\ U_{\Sigma\Lambda} & U_{\Sigma\Sigma} \end{pmatrix}, \quad (3)$$

and  $\mathbf{G}^{(0)}$  is a free Green's function. By the complete Green's function, the inclusive  $K^+$  double-differential lab cross section of  $\Lambda$  production on a nuclear target with a spin  $J_i$  (its  $z$ -component  $M_i$ ) [16] by the one-step mechanism,  $\pi^- p \rightarrow K^+ \Sigma^-$  via  $\Sigma^-$  doorways, is given by

$$\frac{d^2\sigma}{d\Omega_K dE_K} = \beta \frac{1}{[J_i]} \sum_{M_i} (-) \frac{1}{\pi} \text{Im} \sum_{\alpha\alpha'} \langle F_\Sigma^\alpha \dagger G_{\Sigma\Sigma}^{\alpha\alpha'} F_\Sigma^{\alpha'} \rangle, \quad (4)$$

where a production amplitude

$$F_\Sigma^\alpha = \bar{f}_{\pi^- p \rightarrow K^+ \Sigma^-} \chi_{\mathbf{p}_K}^{(-)*} \chi_{\mathbf{p}_\pi}^{(+)} \langle \alpha | \hat{\psi}_p | i \rangle, \quad (5)$$

$[J_i] = 2J_i + 1$ , and the kinematical factor  $\beta$  expresses the translation from the two-body  $\pi^- p$  lab system to the  $\pi^- {}^{10}\text{B}$  lab system.  $\bar{f}_{\pi^- p \rightarrow K^+ \Sigma^-}$  is a Fermi-averaged amplitude for the  $\pi^- p \rightarrow K^+ \Sigma^-$  reaction in nuclear medium, and  $\chi_{\mathbf{p}_K}^{(-)}$  and  $\chi_{\mathbf{p}_\pi}^{(+)}$  are the distorted waves for outgoing  $K^+$  and incoming  $\pi^-$  mesons, respectively, taken into account the recoil effects [18].  $\langle \alpha | \hat{\psi}_p | i \rangle$  is a hole-state wave function for a struck proton in the target, where  $\alpha$  denotes the complete set of eigenstates for the system. The inclusive  $\Lambda$  spectrum in Eq.(4) can be decomposed into different physical processes [16, 17], by using the identity

$$\text{Im}(F_\Sigma^\dagger G_{\Sigma\Sigma} F_\Sigma) = F_\Sigma^\dagger \Omega^{(-)\dagger} (\text{Im} G_\Lambda^{(0)}) \Omega^{(-)} F_\Sigma$$

$$\begin{aligned}
& + F_{\Sigma}^{\dagger} G_{\Sigma\Lambda}^{\dagger} (\text{Im} U_{\Lambda}) G_{\Lambda\Sigma} F_{\Sigma} \\
& + F_{\Sigma}^{\dagger} G_{\Sigma\Sigma}^{\dagger} (\text{Im} U_{\Sigma}) G_{\Sigma\Sigma} F_{\Sigma},
\end{aligned} \tag{6}$$

where  $\Omega^{(-)}$  is the Möller wave operator.

The diagonal (optical) potentials for  $\mathbf{U}$  in Eq. (3) are given by the Woods-Saxon (WS) form:

$$U_Y(r) = (V_Y + iW_Y g(E_{\Lambda})) f(r) \tag{7}$$

for  $Y = \Lambda$  or  $\Sigma^{-}$ , where  $f(r) = [1 + \exp((r - R)/a)]^{-1}$  with  $a = 0.6$  fm,  $r_0 = 1.088 + 0.395A^{-2/3}$  fm and  $R = r_0(A - 1)^{1/3} = 2.42$  fm for the mass number  $A = 10$  [19]. Here we used  $V_{\Lambda} = -30$  MeV for the  ${}^9\text{Li} \otimes \Lambda$  channel, and assumed  $V_{\Sigma} = 0$  MeV to describe the effective  $\Sigma$  state of the  ${}^9\text{Be}^* \otimes \Sigma^{-}$  channel. The spreading imaginary potential,  $\text{Im} U_Y$ , can describe complicated excited-states for  ${}^{10}_{\Lambda}\text{Li}$ ;  $g(E_{\Lambda})$  is an energy-dependent function which linearly increases from 0 at  $E_{\Lambda} = 0$  MeV to 1 at  $E_{\Lambda} = 60$  MeV, as often used in nuclear optical models. The strength parameter  $W_Y$  should be adjusted appropriately to reproduce the data. The coupling  $\Lambda$ - $\Sigma$  potential  $U_{\Sigma\Lambda}$  in off-diagonal parts for  $\mathbf{U}$  is written by

$$\begin{aligned}
U_{\Sigma\Lambda}(r) = & \langle {}^9\text{Be}^* \otimes \Sigma^{-} | \frac{1}{\sqrt{3}} \sum_j v_{\Sigma\Lambda}(\mathbf{r}_j, \mathbf{r}) \boldsymbol{\tau}_j \cdot \boldsymbol{\phi} \\
& \times | {}^9\text{Li} \otimes \Lambda \rangle,
\end{aligned} \tag{8}$$

where  $v_{\Sigma\Lambda}(\mathbf{r}_j, \mathbf{r})$  is a two-body  $\Lambda N$ - $\Sigma N$  potential including the spin-spin interaction, and  $\boldsymbol{\tau}_j$  denotes the  $j$ -th nucleon isospin operator and  $\boldsymbol{\phi}$  is defined as  $|\Sigma\rangle = \boldsymbol{\phi}|\Lambda\rangle$  in isospin space [20]. Here we assumed  $U_{\Sigma\Lambda}(r) = V_{\Sigma\Lambda} f(r)$  in a real potential for simplicity, where  $V_{\Sigma\Lambda}$  is an effective strength parameter. We will attempt to determine the values of  $W_{\Sigma}$  and  $V_{\Sigma\Lambda}$  phenomenologically by fitting to a spectral shape of the experimental data.

For the  ${}^{10}\text{B}(3^{+};0)$  target nucleus, we use single-particle wave functions for a proton, which are calculated by a WS potential [21] with  $V_0^N = -61.36$  MeV fitting to the charge radius of 2.45 fm [22]. Due to large momentum transfer  $q \simeq 270$ -370 MeV/c in the  $(\pi^{-}, K^{+})$  reaction, we simplify the computational procedure for the distorted waves,  $\chi_{\mathbf{p}_K}^{(-)}$  and  $\chi_{\mathbf{p}_{\pi}}^{(+)}$ , with the help of the eikonal approximation. In order to reduce ambiguities in the distorted-waves, we adopt the same parameters used in calculations for the  $\Lambda$  and  $\Sigma^{-}$  quasi-free spectra in nuclear  $(\pi^{\mp}, K^{+})$  reactions [18]. Here we used total cross sections of  $\sigma_{\pi} = 32$  mb for  $\pi^{-}N$  scattering and  $\sigma_K = 12$  mb for  $K^{+}N$  one, and  $\alpha_{\pi} = \alpha_K = 0$ , as the distortion parameters [18]. The Fermi-averaged amplitude  $\bar{f}_{\pi^{-}p \rightarrow K^{+}\Sigma^{-}}$  is obtained by the optimal

Fermi-averaging for the  $\pi^-p \rightarrow K^+\Sigma^-$  t-matrix [18]; we used  $20 \mu\text{b/sr}$  as the lab cross section of  $d\sigma/d\Omega = |\bar{f}_{\pi^-p \rightarrow K^+\Sigma^-}|^2$ .

FIG. 2

Now let us examine the dependence of the spectral shape on two important parameters,  $W_\Sigma$  and  $V_{\Sigma\Lambda}$  by comparing the calculated inclusive  $\Lambda$  spectrum for  $^{10}_\Lambda\text{Li}$  with the data of  $^{10}\text{B}(\pi^-, K^+)$  experiments at KEK [4]. The cross sections of the data are three orders of magnitude less than those for  $^{10}_\Lambda\text{B}$  in  $^{10}\text{B}(\pi^+, K^+)$  reactions. In Fig. 2, we show the calculated spectra by the one-step mechanism at  $p_\pi = 1.20 \text{ GeV/c}$  ( $6^\circ$ ) for the several values of  $-W_\Sigma$  when we use  $V_{\Sigma\Lambda} = 11 \text{ MeV}$  which leads to the  $\Sigma^-$ -mixing probability of  $P_\Sigma = 0.57 \%$  in the  $^{10}_\Lambda\text{Li}$  bound state. We have the peak of the bound state with a  $[0p_{\frac{3}{2}}^{-1}s_{\frac{1}{2}}^\Lambda]_{2-}$  configuration at  $E_\Lambda \simeq -10.0 \text{ MeV}$ , and the peaks of the excited states with  $[0p_{\frac{3}{2}}^{-1}p_{\frac{3}{2},\frac{1}{2}}^\Lambda]_{3+,1+}$  configurations at  $E_\Lambda = 1\text{-}3 \text{ MeV}$ . Since the non-spin-flip processes with the large momentum transfer  $q$  are known to dominate in the  $\pi^-p \rightarrow K^+\Sigma^-$  reaction at  $1.20 \text{ GeV/c}$ , these spin-stretched states are mainly populated. We find that the value of  $-W_\Sigma$  significantly affects a shape of the  $\Lambda$  spectrum for the continuum states which can be populated via  $\Sigma^-p \rightarrow \Lambda n$  processes in  $^9\text{Be}^*$  together with the core-nucleus breakup; this  $\Lambda$  strength mainly arises from a term of  $G_{\Sigma\Sigma}^\dagger(\text{Im}U_\Sigma)G_{\Sigma\Sigma}$  in Eq. (6). We recognize that the calculated spectrum with  $-W_\Sigma = 20\text{-}30 \text{ MeV}$  can reproduce the shape of the data in the continuum region [12], and these values of  $-W_\Sigma$  are consistent with the analysis of  $\Sigma^-$  production by the  $(\pi^-, K^+)$  reactions [18]. Obviously, the parameter  $W_\Sigma$  does not contribute to the spectrum of the bound state. It should be noticed that the contribution of the two-step processes in the continuum spectrum is rather small, as the dashed curves shown in Fig. 2.

FIG. 3

On the other hand, the  $\Lambda$ - $\Sigma$  coupling potential plays an essential role in the formation of the  $\Lambda$  bound state. In Fig. 3, we show the dependence of the cross section for the bound state in  $^{10}_\Lambda\text{Li}$  on the values of  $V_{\Sigma\Lambda}$  when  $-W_\Sigma = 20 \text{ MeV}$ . We find that the calculated spectrum for the bound state is quite sensitive to  $V_{\Sigma\Lambda}$ ; when we use  $V_{\Sigma\Lambda} = 4, 8, 10, 11$  and  $12 \text{ MeV}$ , the probabilities of the  $\Sigma^-$  admixture in the  $2^-$  bound state are  $P_{\Sigma^-} = 0.075, 0.30, 0.47, 0.57$  and  $0.68 \%$ , respectively. The positions of the peaks for  $^{10}_\Lambda\text{Li}$  are slightly shifted downward by  $\Delta E_\Lambda \simeq -\langle U_{\Sigma\Lambda} \rangle^2 / \Delta M \simeq -\Delta M \cdot P_{\Sigma^-}$ , e.g.,  $-456 \text{ keV}$  for  $V_{\Sigma\Lambda} = 11$

MeV, which is about 4-5 times larger than that of  ${}^7_\Lambda\text{Li}$  [23]. For the order of  $V_{\Sigma\Lambda} = 10\text{-}12$  MeV ( $P_{\Sigma^-} = 0.47\text{-}0.68\%$ ), the calculated spectra can fairly reproduce the data, whereas it is not appropriate to a detailed study of the structure of  ${}^{10}_\Lambda\text{Li}$  because of the simple single-particle picture we adopted here. Such a  $\Sigma^-$  admixture seems to be consistent with the recent microscopic calculations [7, 8, 24]. This consistency of  $V_{\Sigma\Lambda}$  considerably enhances the reliability of our calculations. Consequently, the calculated spectrum by the one-step mechanism explains the  ${}^{10}\text{B}(\pi^-, K^+)$  data. This fact implies that the one-step mechanism dominates in the  $(\pi^-, K^+)$  reaction, and our calculations provide a capability of extracting a production mechanism from the data of this reaction. Some discrepancy between the results and the data in the bound-state region surrounding  $E_\Lambda \simeq -5$  MeV might be improved by a sophisticated shell-model calculation with configuration-mixing [24].

### ———— TABLE I ————

The early theoretical prediction by Tretyakova and Lansky [14] differs from the present result. They have shown that the  $\Sigma^-$ -mixing probabilities on  $p$ -shell nuclei are the order of  $10^{-3}\text{-}10^{-2}\%$ , which are smaller than ours by one or more orders of magnitude, within Hartree-Fock single-particle calculations based on two-body  $\Lambda N$ - $\Sigma N$  effective interactions [7]. The  $\Lambda$ - $\Sigma$  coupling in the Hartree-Fock states seems to be hindered by the lack of active configurations. For the two-step mechanism,  $\pi^- p \rightarrow \pi^0 n$  followed by  $\pi^0 p \rightarrow K^+ \Lambda$  or  $\pi^- p \rightarrow K^0 \Lambda$  followed by  $K^0 p \rightarrow K^+ n$ , in the DCX  ${}^{10}\text{B}(\pi^-, K^+)$  reaction, we roughly estimate the integrated lab cross sections of  $d\sigma/d\Omega$  for the  ${}^{10}_\Lambda\text{Li}$  bound state with a harmonic oscillator model in the eikonal approximation [25]. In Table I, we show that the calculated value of  $d\sigma/d\Omega$  at 1.20 GeV/c ( $6^\circ$ ) by the two-step mechanism is rather small (1-2 nb/sr) compared to that by the one-step one. (See also Fig. 2.) The incident-momentum dependence of  $d\sigma/d\Omega$  in the data is similar to that in the one-step mechanism. Therefore, we believe that the one-step mechanism is dominant in the  $(\pi^-, K^+)$  reaction at 1.20 GeV/c. The  ${}^{10}\text{B}(\pi^-, K^+)$  experiment at KEK [12] might be interpreted as a measurement of the  $\Sigma^-$  admixture in the  $\Lambda$  hypernucleus. The  $\Sigma^-$  admixture gives a key for understanding of the EoS and neutron stars [9].

In conclusion, the calculated spectrum of the  ${}^{10}_\Lambda\text{Li}$  hypernucleus by the one-step mechanism via  $\Sigma^-$  doorways fully explains the data of the DCX  ${}^{10}\text{B}(\pi^-, K^+)$  reaction at 1.20 GeV/c, rather than by the two-step mechanism. The result shows that the  $\Sigma^-$  admixture

probability in the  $^{10}_{\Lambda}\text{Li}$  bound state is the order of  $10^{-1}$  %. The sensitivity to the potential parameters implies that the nuclear ( $\pi^-$ ,  $K^+$ ) reactions with much less background experimentally provide a high ability for the theoretical analysis of precise wave functions in the neutron-rich  $\Lambda$  hypernuclei. The detailed analysis based on microscopic nuclear calculations is required for forthcoming J-PARC experiment [13]. This investigation is in progress.

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- [1] For example, A. Gal and R.S. Hayano, (Eds.), Special Issue on Recent Advances in Strangeness Nuclear Physics, Nucl. Phys. A804 (2008).
  - [2] S. Balberg and A. Gal, Nucl. Phys. A625, 435 (1997); M. Baldo, G.F. Burgio and H.-J. Schulze, Phys. Rev. C58, 3688 (1998), and references therein.
  - [3] E. Friedman and A. Gal, Phys. Rep. 452, 89 (2007).
  - [4] P.K. Saha et al., Phys. Rev. C70, 044613 (2004).
  - [5] Y. Fujiwara, Y. Suzuki and C. Nakamoto, Prog. Part. Nucl. Phys. 58, 439 (2007), and references therein.
  - [6] L. Maijling, Nucl. Phys. A585, 211c (1995).
  - [7] Y. Akaishi, T. Harada, S. Shinmura and Khin Swe Myint, Phys. Rev. Lett. 84, 3539 (2000).
  - [8] H. Nemura and Y. Akaishi, Nucl. Phys. A738, 254 (2004).
  - [9] Khin Swe Myint, T. Harada, S. Shinmura and Y. Akaishi, Few-Body System Suppl. 12, 383 (2000); S. Shinmura, Khin Swe Myint, T. Harada and Y. Akaishi, J. Phys. G28, 1 (2002).
  - [10] K. Kubota et al., Nucl. Phys. A602, 327 (1996).
  - [11] M. Agnello et al., Phys. Lett. B640, 145 (2006).
  - [12] P.K. Saha et al., Phys. Rev. Lett. 94, 052502 (2005).
  - [13] A. Sakaguchi et al., “*Production of Neutron-Rich  $\Lambda$ -Hypernuclei with the Double Charge-Exchange Reactions*”, Proposal for Nuclear and Particle Physics experiments at the J-PARC (2007), [http://j-parc.jp/NuclPart/Proposal\\_e.html](http://j-parc.jp/NuclPart/Proposal_e.html)
  - [14] T. Yu. Tretyakova and D. E. Lanskoj, Phys. At. Nucl. 66 (2003) 1651; Nucl. Phys. A691, 51c



- (2001).
- [15] H. Bandō, T. Motoba and J. Žofka, Int. J. Mod. Phys. A5, 4021 (1990).
  - [16] T. Harada, Phys. Rev. Lett. 81, 5287 (1998); Nucl. Phys. A672, 181 (2000).
  - [17] O. Morimatsu and K. Yazaki, Prog. Part. Nucl. Phys. 33, 679 (1994), and references therein.
  - [18] T. Harada and Y. Hirabayashi, Nucl. Phys. A744, 323 (2004); Nucl. Phys. A759, 143 (2005).
  - [19] D.J. Millener, C.B. Dover and A. Gal, Phys. Rev. C38, 2700 (1988).
  - [20] N. Auerbach, Phys. Rev. C35, 1798 (1987).
  - [21] A. Bohr and M. Mottelson, *Nuclear structure*, Vol. 1 (Benjamin, New York, 1969) p.238.
  - [22] H. de Vries et al., At. Data Nucl. Tables 36, 459 (1987).
  - [23] D. J. Millener, Springer Lecture Notes in Physics 724, 31 (2007); D. J. Millener, Nucl. Phys. A804, 84 (2008).
  - [24] A. Umeya and T. Harada, arXiv:0810.4591 [nucl-th] (submitted for publication).
  - [25] R.E. Chrien, C.B. Dover and A. Gal, Czech. J. Phys., 42, 1089 (1992).

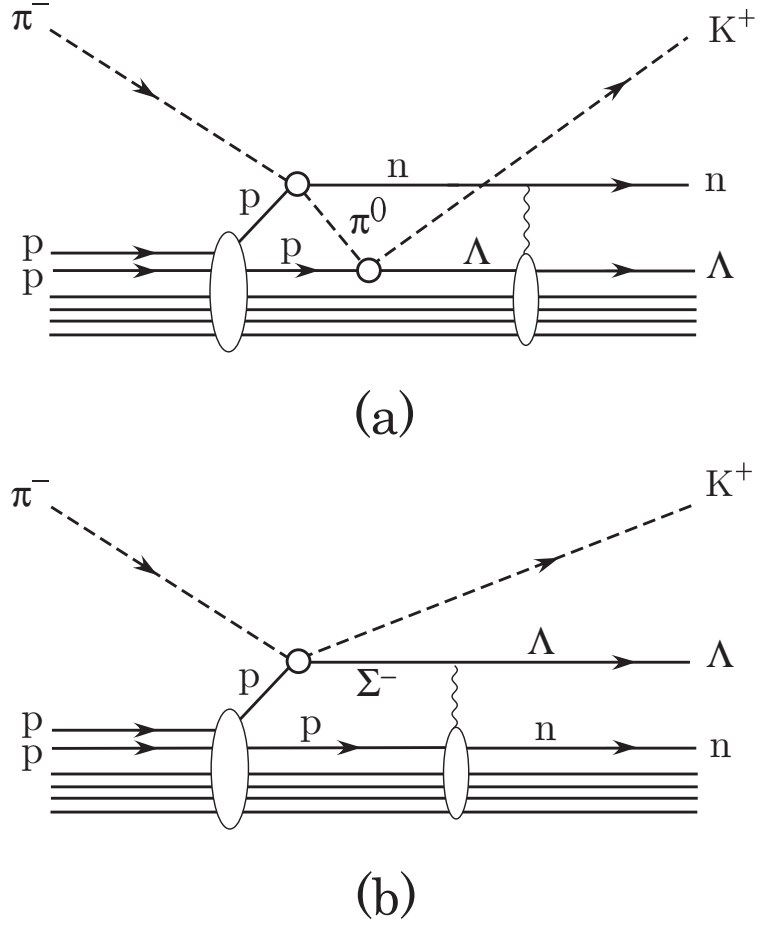


FIG. 1: Diagrams for DCX nuclear ( $\pi^-$ ,  $K^+$ ) reactions, leading to production of  $\Lambda$  hypernuclear states: (a) a two-step mechanism,  $\pi^- p \rightarrow \pi^0 n$  followed by  $\pi^0 p \rightarrow K^+ \Lambda$ , and (b) a one-step mechanism,  $\pi^- p \rightarrow K^+ \Sigma^-$  via  $\Sigma^-$  doorways caused by the  $\Sigma^- p \leftrightarrow \Lambda n$  coupling.

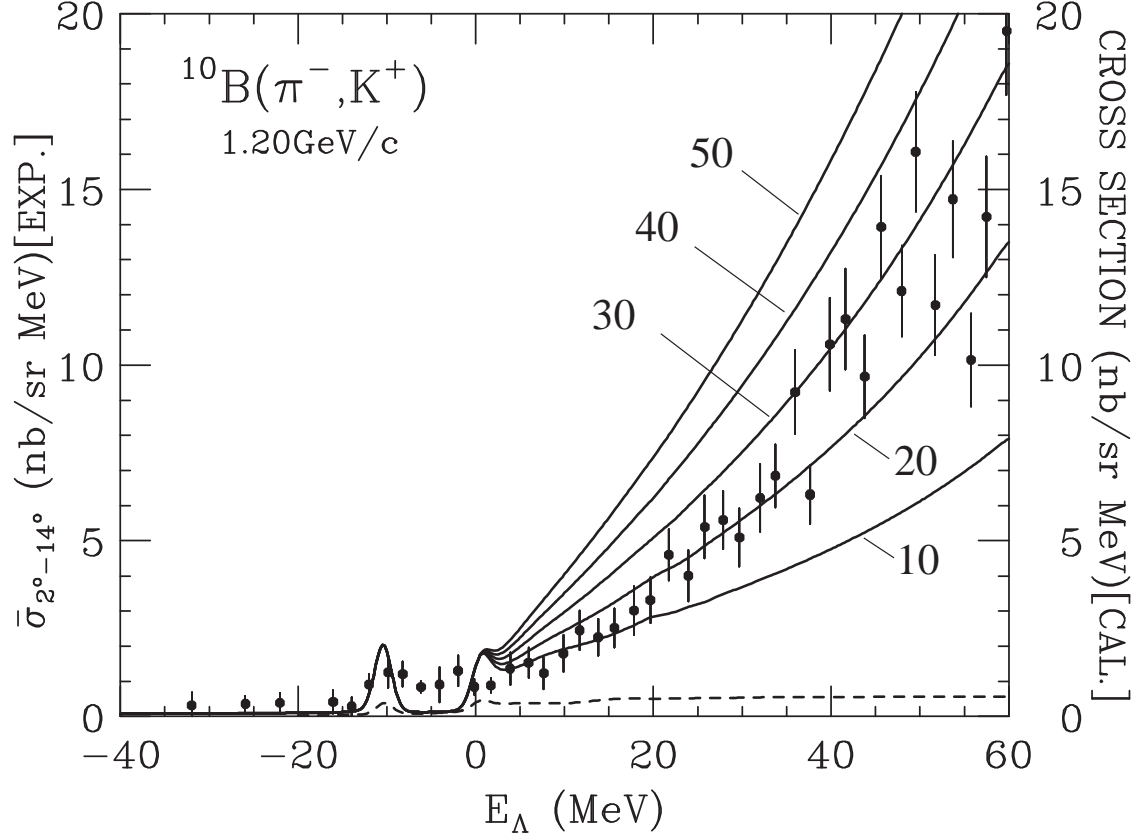


FIG. 2: Calculated inclusive  $\Lambda$  spectra by the one-step mechanism in the  $^{10}\text{B}(\pi^-, K^+)$  reaction at  $p_\pi = 1.20 \text{ GeV}/c$  ( $6^\circ$ ), together with the experimental data [12]. The solid curves denote the  $K^+$  spectra by  $-W_\Sigma = 10, 20, 30, 40$  and  $50 \text{ MeV}$  when  $V_{\Sigma\Lambda} = 11 \text{ MeV}$  ( $P_\Sigma = 0.57 \%$ ), with a detector resolution of  $2.5 \text{ MeV}$  FWHM. The dashed curve denotes the inclusive  $\Lambda$  spectrum by the two-step mechanism.

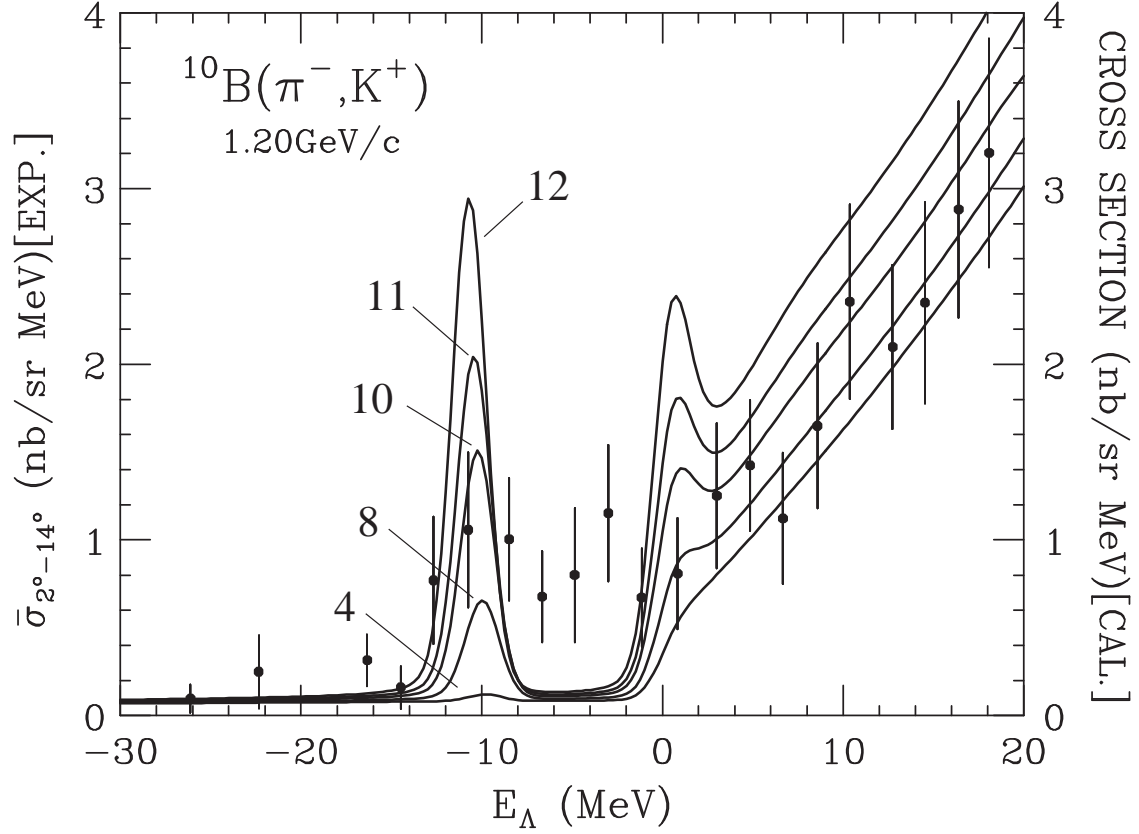


FIG. 3: Calculated inclusive  $\Lambda$  spectra by the one-step mechanism near the  $\Lambda$ -threshold in the  $^{10}\text{B}(\pi^-, K^+)$  reaction at 1.20 GeV/c ( $6^\circ$ ), by changing  $V_{\Sigma\Lambda}$  for the  $\Lambda$ - $\Sigma$  coupling potential. The experimental data are taken from Ref. [12]. The solid curves denote for  $V_{\Sigma\Lambda} = 4, 8, 10, 11$  and 12 MeV when  $-W_\Sigma = 20$  MeV, with a detector resolution of 2.5 MeV FWHM.

TABLE I: Calculated results of the integrated lab cross sections of  $d\sigma/d\Omega$  for the  $^{10}_{\Lambda}\text{Li } 2^-$  bound state with two-step and one-step processes in  $^{10}\text{B}(\pi^-, K^+)$  reactions at  $6^\circ$ , compared with the data [12]. The value in the bracket is a lower limit one with  $\Lambda$  quasi-free corrections.

$p_\pi$ (GeV/c)	Two-step <sup>a</sup> (nb/sr)	One-step <sup>b</sup> (nb/sr)	Exp. [12] (nb/sr)
1.05	$\sim 1.6$	2.4	$5.8 \pm 2.2^c$
1.20	$\sim 1.2$	5.4	$11.3 \pm 1.9^c$ ( $9.6 \pm 2.0$ )

<sup>a</sup>Sum of the cross sections via  $\pi^- p \rightarrow \pi^0 n$  followed by  $\pi^0 p \rightarrow K^+ \Lambda$  and  $\pi^- p \rightarrow K^0 \Lambda$  followed by  $K^0 p \rightarrow K^+ n$ , by a simple harmonic oscillator model.

<sup>b</sup> $P_{\Sigma^-} = 0.57\%$  ( $V_{\Sigma\Lambda} = 11$  MeV) is assumed.

<sup>c</sup>All the events for  $-20 \text{ MeV} \leq E_\Lambda \leq 0 \text{ MeV}$ .